

# Metropolis-Hastings

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Programming for Scientists

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# Random Hill Climbing

- 1  $x_0 \leftarrow \text{random}$
- 2 For  $t \in \{1 \dots N\}$ 
  - a  $c \leftarrow x_{t-1} + \mathcal{N}(\mathbf{0}, \sigma)$
  - b If  $f(c) < f(x_{t-1})$ ,  $x_t \leftarrow c$
  - c Else,  $x_t \leftarrow x_{t-1}$

# Random Numbers

Many algorithms need random numbers.

# Pseudo-Random Numbers

PRN are deterministically generated numbers that look like random.

# Pseudo Random Number Example

$$x_{n+1} = 1103515245x_n + 12345 \pmod{2^{32}}$$

# Pseudo Random Sequences

Each number is generated based on the previous.

# Seeding the Sequence

```
import random
random.seed(0)
print random.random()
print random.random()
print random.random()
print
```

```
random.seed(0)
print random.random()
print random.random()
print random.random()
```

prints

```
0.844421851525
0.75795440294
0.420571580831
```

```
0.844421851525
0.75795440294
0.420571580831
```

## Seeding the Sequence (II)

```
import numpy.random
numpy.random.seed(0)
print numpy.random.random()
print numpy.random.random()
print numpy.random.random()
print
```

prints

```
0.548813503927
0.715189366372
0.602763376072
```

```
numpy.random.seed(0)
print numpy.random.random()
print numpy.random.random()
print numpy.random.random()
```

```
0.548813503927
0.715189366372
0.602763376072
```

# Sampling From a Distribution

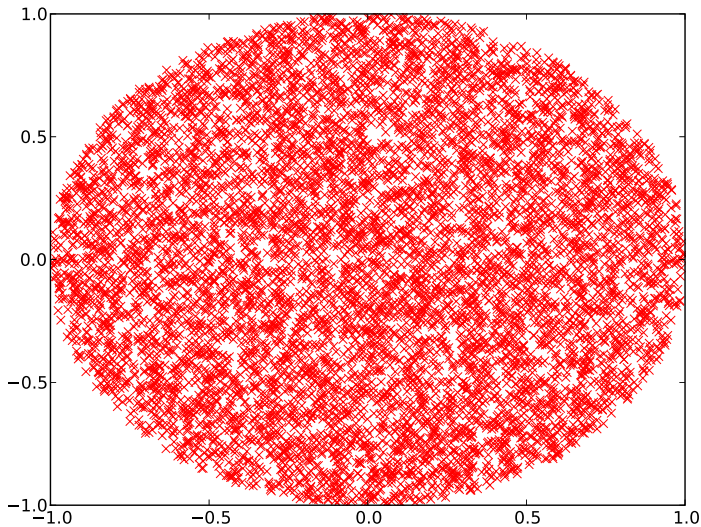
Given a random number uniformly distributed on  $[0, 1]$ ,  
how do we sample from  $p(x)$ ?

Check out what's already written (scipy.stats has plenty).

## Scipy.stats

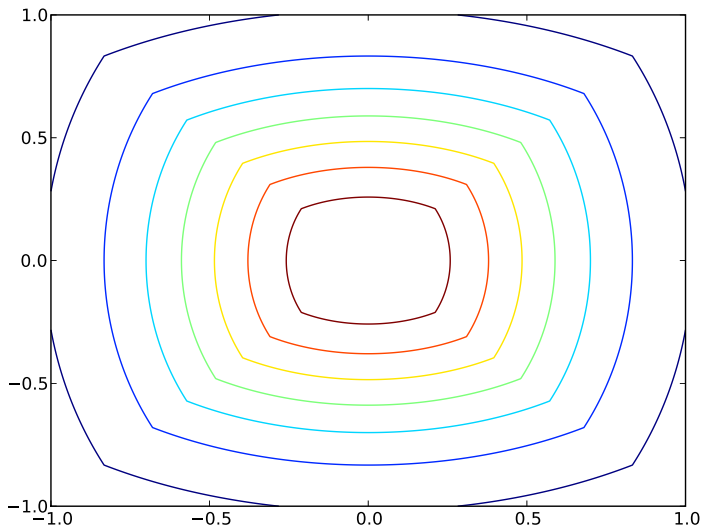
- Normal
- Exponential
- Poisson
- ...

# How do I Sample a sphere?



What if  $P(x)$  is really complicated?

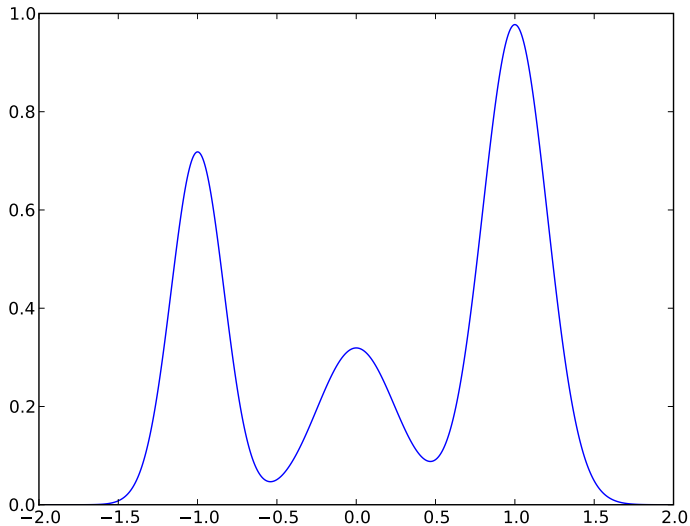
# Example



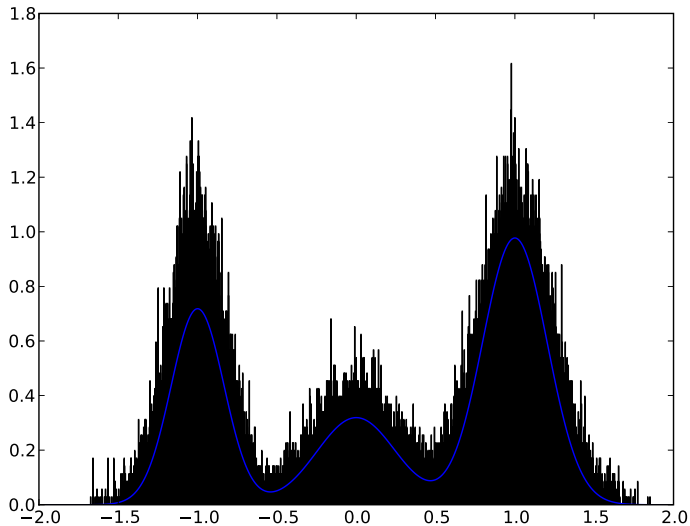
# Metropolis-Hastings

- $x_0 \leftarrow$  random
- For  $t \in \{1 \dots T\}$ :
  - 1  $c \leftarrow$  sample from  $Q(x'|x_t)$
  - 2  $a \leftarrow \frac{P(c)Q(x_t|c)}{P(x_t)Q(c|x_t)}$
  - 3  $r \leftarrow U(0, 1)$
  - 4 If  $r < a$  :  $x_{t+1} \leftarrow c$
  - 5 Else  $x_{t+1} \leftarrow x_t$

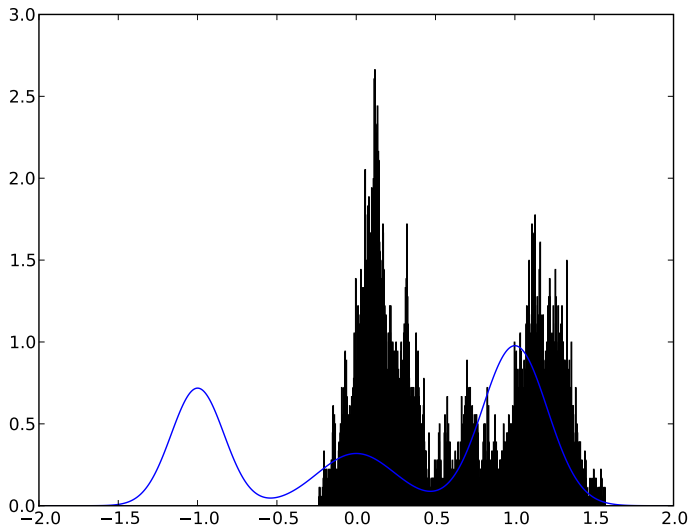
# Metropolis-Hastings Example (Simple)



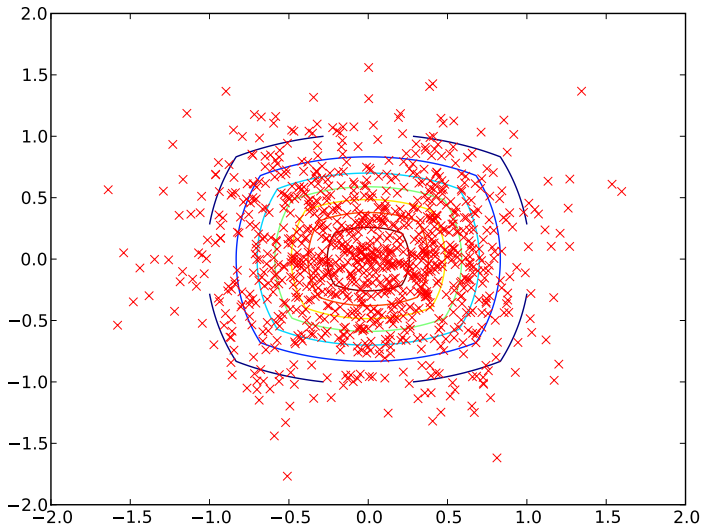
# Metropolis-Hastings Example



# Metropolis-Hastings Example

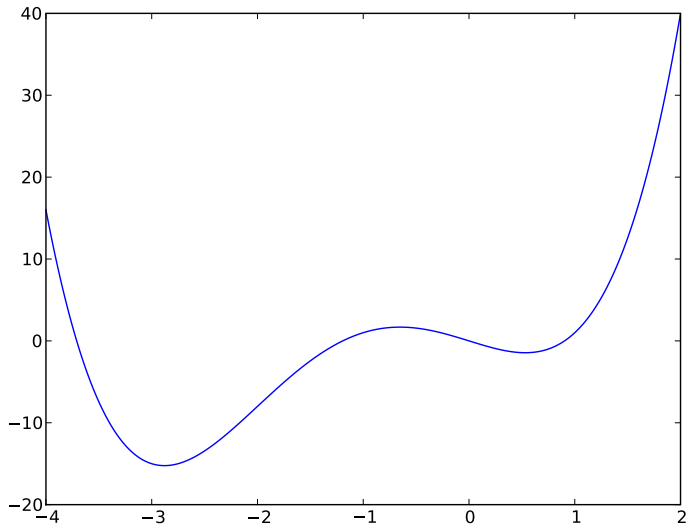


# Metropolis-Hastings More Complex Example

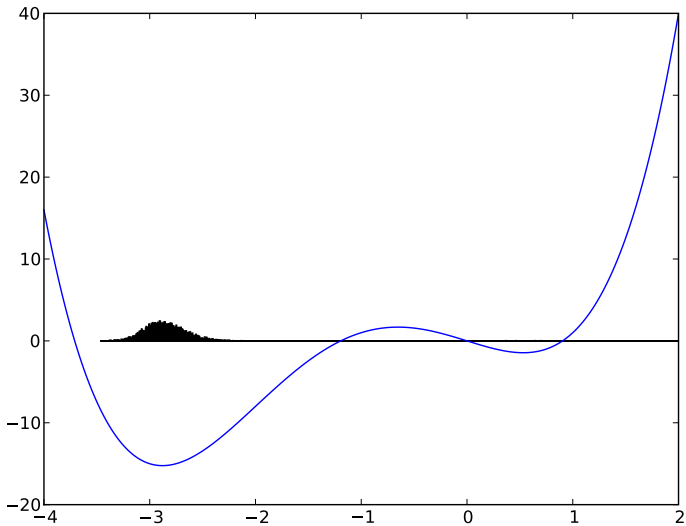


$$P(x) = k \exp(-\beta f(x))$$

# Minimisation



# Minimisation



# Simulated Annealing Trick

$$P(x)/P(x') = \exp (T(f(x) - f(x')))$$

# Simulated Annealing Trick

$$P(x)/P(x') = \exp (T(f(x) - f(x')))$$

Start hot, reduce the temperature.

# Simulated Annealing

Starts like Metropolis-Hastings, finished like Random Hill Climbing.